STD. VI

SUBTRACTION OF ALGEBRAIC EXPRESSIONS

Rule *Change the sign of each term of the expression to be subtracted and add to the expression from which subtraction is to be made.*

EXAMPLE 4. *Subtract* $6xy - 4x^2 - y^2 - 2$ *from* $x^2 - 3xy + 7y^2 + 5$.

Solution Arranging the like terms columnwise, changing the sign of each term of the expression to be subtracted and then adding, we get

$$\begin{array}{r} x^2 - 3xy + 7y^2 + 5 \\ -4x^2 + 6xy - y^2 - 2 \quad \text{(change the sign of each term and add)} \\ + \quad - \quad + \quad + \\ \hline 5x^2 - 9xy + 8y^2 + 7 \end{array}$$

EXAMPLE 5. *From the sum of* $6x^4 - 3x^3 + 7x^2 - 5x + 1$ *and* $-3x^4 + 5x^3 - 9x^2 + 7x - 2$ *subtract* $2x^4 - 5x^3 + 2x^2 - 6x - 8$.

Solution We have:

$$\begin{array}{rr} & 6x^4 - 3x^3 + 7x^2 - 5x + 1 \\ \text{Plus} & -3x^4 + 5x^3 - 9x^2 + 7x - 2 \\ \hline & 3x^4 + 2x^3 - 2x^2 + 2x - 1 \\ \text{Minus} & 2x^4 - 5x^3 + 2x^2 - 6x - 8 \\ & - \quad + \quad - \quad + \quad + \\ \hline & x^4 + 7x^3 - 4x^2 + 8x + 7 \end{array}$$

EXERCISE 8C

1. Add:
   (i) $3x, 7x$
   (ii) $7x, -9y$
   (iii) $2xy, 5xy, -xy$
   (iv) $3x, 2y$
   (v) $2x^2, -3x^2, 7x^2$
   (vi) $7xyz, -5xyz, 9xyz, -8xyz$
   (vii) $6a^3, -4a^3, 10a^3, -8a^3$
   (viii) $x^2 - a^2, -5x^2 + 2a^2, -4x^2 + 4a^2$

2. Add the following:

   (i) $x - 3y - 2z$
   $5x + 7y - z$
   $-7x - 2y + 4z$

   (ii) $m^2 - 4m + 5$
   $-2m^2 + 6m - 6$
   $-m^2 - 2m - 7$

   (iii) $2x^2 - 3xy + y^2$
   $-7x^2 - 5xy - 2y^2$
   $4x^2 + xy - 6y^2$

   (iv) $4xy - 5yz - 7zx$
   $-5xy + 2yz + zx$
   $-2xy - 3yz + 3zx$

3. Add:
   (i) $3a - 2b + 5c, 2a + 5b - 7c, -a - b + c$
   (ii) $8a - 6ab + 5b, -6a - ab - 8b, -4a + 2ab + 3b$

(iii) $2x^3 - 3x^2 + 7x - 8$, $-5x^3 + 2x^2 - 4x + 1$, $3 - 6x + 5x^2 - x^3$
(iv) $2x^2 - 8xy + 7y^2 - 8xy^2$, $2xy^2 + 6xy - y^2 + 3x^2$, $4y^2 - xy - x^2 + xy^2$
(v) $x^3 + y^3 - z^3 + 3xyz$, $-x^3 + y^3 + z^3 - 6xyz$, $x^3 - y^3 - z^3 - 8xyz$
(vi) $2 + x - x^2 + 6x^3$, $-6 - 2x + 4x^2 - 3x^3$, $2 + x^2$, $3 - x^3 + 4x - 2x^2$

4. Subtract:
(i) $5x$ from $2x$ (ii) $-xy$ from $6xy$ (iii) $3a$ from $5b$
(iv) $-7x$ from $9y$ (v) $10x^2$ from $-7x^2$ (vi) $a^2 - b^2$ from $b^2 - a^2$

5. Subtract:
(i) $5a + 7b - 2c$ from $3a - 7b + 4c$
(ii) $a - 2b - 3c$ from $-2a + 5b - 4c$
(iii) $5x^2 - 3xy + y^2$ from $7x^2 - 2xy - 4y^2$
(iv) $6x^3 - 7x^2 + 5x - 3$ from $4 - 5x + 6x^2 - 8x^3$
(v) $x^3 + 2x^2y + 6xy^2 - y^3$ from $y^3 - 3xy^2 - 4x^2y$
(vi) $-11x^2y^2 + 7xy - 6$ from $9x^2y^2 - 6xy + 9$
(vii) $-2a + b + 6d$ from $5a - 2b - 3c$

6. Simplify:
(i) $2p^3 - 3p^2 + 4p - 5 - 6p^3 + 2p^2 - 8p - 2 + 6p + 8$
(ii) $2x^2 - xy + 6x - 4y + 5xy - 4x + 6x^2 + 3y$
(iii) $x^4 - 6x^3 + 2x - 7 + 7x^3 - x + 5x^2 + 2 - x^4$

7. From the sum of $3x^2 - 5x + 2$ and $-5x^2 - 8x + 6$, subtract $4x^2 - 9x + 7$.

8. If $A = 7x^2 + 5xy - 9y^2$, $B = -4x^2 + xy + 5y^2$ and $C = 4y^2 - 3x^2 - 6xy$ then show that $A + B + C = 0$.

9. What must be added to $5x^3 - 2x^2 + 6x + 7$ to make the sum $x^3 + 3x^2 - x + 1$?

10. Let $P = a^2 - b^2 + 2ab$, $Q = a^2 + 4b^2 - 6ab$, $R = b^2 + 6$, $S = a^2 - 4ab$ and $T = -2a^2 + b^2 - ab + a$. Find $P + Q + R + S - T$.

11. What must be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$?

12. How much is $a + 2b - 3c$ greater than $2a - 3b + c$?

13. How much less than $x - 2y + 3z$ is $2x - 4y - z$?

14. By how much does $3x^2 - 5x + 6$ exceed $x^3 - x^2 + 4x - 1$?

15. Subtract the sum of $5x - 4y + 6z$ and $-8x + y - 2z$ from the sum of $12x - y + 3z$ and $-3x + 5y - 8z$.

16. By how much is $2x - 3y + 4z$ greater than $2x + 5y - 6z + 2$?

17. By how much does 1 exceed $2x - 3y - 4$?

USE OF GROUPING SYMBOLS

When we make operations on two or more algebraic expressions, we separate them by the symbols of groupings, namely, parentheses ( ), braces { } and brackets [ ].

In simplifying such expressions, we first remove the grouping symbols, using the laws given below:

(i) If a '+' sign precedes a symbol of grouping, the grouping symbol may be removed without any change in the sign of the terms.

(ii) If a '–' sign precedes a symbol of grouping, the grouping symbol may be removed and the sign of each term is changed.

(iii) If more than one grouping symbol is present in an expression, we remove the innermost grouping symbol first and combine the like terms, if any. We continue the process outwards until all the grouping symbols have been removed.

EXAMPLE 1. *Simplify:* $(a^2 - 8ab - 5) + (3ab - 4a^2 + 8)$

Solution Clearly, a '+' sign precedes the second parenthesis, so we remove it without changing the signs of the terms within it.

$$\begin{aligned}\therefore\ & (a^2 - 8ab - 5) + (3ab - 4a^2 + 8)\\ &= a^2 - 8ab - 5 + 3ab - 4a^2 + 8\\ &= (1 - 4)a^2 + (-8 + 3)ab + (-5 + 8)\\ &= -3a^2 - 5ab + 3.\end{aligned}$$

EXAMPLE 2. *Simplify:* $(x^2 - y^2 + 2xy + 1) - (x^2 + y^2 + 4xy - 5)$

Solution Here, a '–' sign precedes the second parenthesis, so we remove it and change the sign of each term within.

$$\begin{aligned}\therefore\ & (x^2 - y^2 + 2xy + 1) - (x^2 + y^2 + 4xy - 5)\\ &= x^2 - y^2 + 2xy + 1 - x^2 - y^2 - 4xy + 5\\ &= (1 - 1)x^2 - 2y^2 + (2 - 4)xy + 6\\ &= -2y^2 - 2xy + 6.\end{aligned}$$

EXAMPLE 3. *Simplify:* $2x - [3y - \{2x - (y - x)\}]$

Solution We first remove the innermost grouping symbol ( ), then { }, and then [ ]. Thus, we have:

$$\begin{aligned}& 2x - [3y - \{2x - (y - x)\}]\\ &= 2x - [3y - \{2x - y + x\}]\\ &= 2x - [3y - \{3x - y\}]\\ &= 2x - [3y - 3x + y]\\ &= 2x - [4y - 3x]\\ &= 2x - 4y + 3x\\ &= 5x - 4y.\end{aligned}$$

EXAMPLE 4. *Simplify:* $2a - [3b - \{a - (2c - 3b) + 4c - 3(a - b - 2c)\}]$

Solution We first remove the innermost grouping symbol ( ), then { }, and then [ ]. Thus, we have:

$$\begin{aligned}& 2a - \big[3b - \{a - (2c - 3b) + 4c - 3(a - b - 2c)\}\big]\\ &= 2a - [3b - \{a - 2c + 3b + 4c - 3a + 3b + 6c\}]\\ &= 2a - [3b - \{-2a + 6b + 8c\}]\\ &= 2a - [3b + 2a - 6b - 8c]\\ &= 2a - [-3b + 2a - 8c]\\ &= 2a + 3b - 2a + 8c\\ &= 3b + 8c.\end{aligned}$$

EXERCISE 8D

Simplify:

1. $a - (b - 2a)$
2. $4x - (3y - x + 2z)$
3. $(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$
4. $-3(a + b) + 4(2a - 3b) - (2a - b)$
5. $-4x^2 + \{(2x^2 - 3) - (4 - 3x^2)\}$
6. $-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$
7. $a - [2b - \{3a - (2b - 3c)\}]$
8. $-x + [5y - \{x - (5y - 2x)\}]$

9. $86 - [15x - 7(6x - 9) - 2\{10x - 5(2 - 3x)\}]$
10. $12x - [3x^3 + 5x^2 - \{7x^2 - (4 - 3x - x^3) + 6x^3\} - 3x]$
11. $5a - [a^2 - \{2a(1 - a + 4a^2) - 3a(a^2 - 5a - 3)\}] - 8a$
12. $3 - [x - \{2y - (5x + y - 3) + 2x^2\} - (x^2 - 3y)]$
13. $xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}]$
14. $2a - 3b - [3a - 2b - \{a - c - (a - 2b)\}]$
15. $-a - [a + \{a + b - 2a - (a - 2b)\} - b]$
16. $2a - [4b - \{4a - (3b - \overline{2a + 2b})\}]$
17. $5x - [4y - \{7x - (3z - 2y) + 4z - 3(x + 3y - 2z)\}]$

Things to Remember

1. *Algebra is generalized arithmetic.*
2. *Letters used to represent numbers are called literals.*
3. *The literals obey all the rules and signs of addition, subtraction, multiplication and division.*
4. $3 \times x = 3x, 1 \times x = x, x \times y = xy, x \times 5 = 5x$, *etc.*
5. $x \times x \times \ldots\ldots$ *5 times* $= x^5$, $y \times y \times y \times \ldots\ldots$ *10 times* $= y^{10}$.
6. *In* x^4, *we call 4 the exponent or index, and x is called the base.*
7. *A symbol having a fixed numerical value is called a constant.*
8. *A symbol which takes on various numerical values is called a variable.*
9. *A combination of constants and variables using any of the signs + and −, or a combination of the signs +, −, × and ÷, is called an algebraic expression.*
10. *'+' or '−' signs separate the expression into various parts, each part is known as a term.*
11. *An expression is called a monomial, a binomial, a trinomial or a quadrinomial if it contains one term, two terms, three terms or four terms respectively.*
12. *In 4x, we have 4 as the numerical factor and x as the literal factor.*
13. *The terms having the same literal factors are called like terms; otherwise they are called unlike terms.*
14. *The sum of several like terms is another like term whose coefficient is the sum of the coefficients of the like terms.*
15. *To subtract an expression from another we change the sign of each term of the expression to be subtracted and then add the two expressions.*
16. *When a grouping symbol preceded by a '−' sign is removed, the sign of each term in the grouping is changed.*